First Quarterly Report on Contract NOw 62 - 0770 - c

62·4-1

Prepared under Bureau of Naval Weapons

Contract NOw 62 - 0770 - c

DEVELOPMENT LABORATORIES INCORPORATED

FIRST QUARTERLY REPORT ON CONTRACT

NOw 62 - 0770 - c

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I. INTRODUCTION

SCOPE OF THE CONTRACT

1.1   The purpose of this contract is the study and development of an angular rate indicating instrument based on measuring the angular rate of an artificially induced vortex. The measurement of the change in rate of the artificially induced vortex is the measure of an externally superimposed rotation on the system. The successful achievement of this development would provide an instrument to replace gyroscopes.

PHASES OF THE CONTRACT

1.2   The scope of the work in Contract NOw 62 – 0770 – c was written in three phases to insure the orderly conduct of the project. They read as follows:

Phase A – Conduct investigations limited to veri^f ·ation of practical methods of measuring the rate of change of gas ve   cities necessary to successful operation of a vortex gyroscope.

Phase B – Conduct investigations limited to verification of practical methods of providing constant initial gas parameters neces- sary to successful operation of a vortex gyroscope.

Phase C – Complete work proposed in Development Laboratories proposals N1.   The contractor shall limit research and development effort initially to Phase A until given written instructions by the technical director to proceed to Phase B.   Similar instructions shall be obtained before proceeding from Phase B. to Phase C.

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II. WORK ACCOMPLISHED DURING FIRST QUARTER

ORGANIZATION OF THE WORK

2.1   Activities on Contract NOw 62 - 0770 - c started the 15th
of March, 1962.   This report covers the work accomplished by the
15th of June, 1962.   Figure 1   shows the organization of the work
in bar chart form.   The work carried out during the first quarter
followed this chart closely.

STUDIES ON MIXED ION SYSTEMS

2.2   The studies carried out on mixed ion systems are presented
in Appendix A.   The studies performed to date have lead to the
following conclusions:

> The heavy ion diffusion rates dictate an environ-
> ment near or above normal gas density.   At signifi-
> cantly lower density, the diffusion rates become
> large and the pulse or wave shape of the group is
> rapidly distorted.

> If mixed ion systems (heavy positive ions plus
> electrons) are considered, then recombination rates
> at normal gas density are high and it is difficult,
> maybe impossible, to retain a measurable ion group
> at the down-stream observation point.   This obser-
> vation, derived from studies of Xe ions in a gas of normal
> density, is less true of a helium system at 10 atmospheres.

> It cannot be concluded from the work done that systems
> with mixed ions cannot be made to work.   However,
> more promising concepts have arisen which are not
> subject to the recombination limitation.   The exploration
> of these concepts appears to be more fruitful than a
> continued study of the mixed ion systems.

STUDY ON SINGLE ION SYSTEMS

2.3   The studies carried out on the diffusion of single ion systems are presented in Appendix B.   The studies performed to date have lead to the general conclusion that the single ion system does not have the inherent limitation imposed by recombination in the mixed ion case, and that a system based on this could be successfully developed.

STUDIES ON THERMAL SYSTEMS

2.4   The studies carried out on the thermal system are presented in Appendix C.   The thermal system consists of pulsed heater and a down-stream detector which senses the arrival of pulses of heated gas.   The detector may be a thermocouple or a resistance thermometer.   The heater can be a simple thin wire.   The studies performed have lead to the general conclusion that this system should be far the easiest of development and that it can be made to work adequately.

THE GAS VELOCITY MEASURING SETUP

2.5   A test setup was designed and constructed during the three months period being reported.   This setup consists of equipment developed for regulating the passage of the working fluid in a test section wherein various electrode systems can be placed for testing.   This test setup is being used in conjunction with a cathode ray oscilloscope and associated electronic components for the visualization of the signals transmitted and received by the electrode system under test.

TEST RESULTS

2.6   After the usual initial difficulties in getting a system functioning, useful results on the measurement on the working fluid velocity are being obtained.   These results are being used to confirm theoretical expected values and establish threshold of operation of measuring electrode systems with various working fluids.   These tests will continue during the next quarter.

III.   THE WORK PLAN

THE WORK PLAN

3.1   Since the problem associated with Phase A of the work statement is being satisfactorily solved and since the working fluid pump problem has been circumvented by a mechanical pump in place of the original method suggested in our proposal, the work plan has been modified to the one shown in Figure 2, which is self-explanatory.

EXPECTED WORK TO BE ACCOMPLISHED DURING NEXT QUARTER

3.2   By the end of the next quarter, we hope to accomplish most of the theoretical work associated with the signal generation and detection and be well on our way into the motion analysis study of a complete angular rate indicator, as well as having considered various alternatives relating to the electrode geometry and space. We will hope to have finished all flow velocity measurements that are required and that the testing of an angular rate instrument be under way.   This of course means that the design of an instrument be also completed by then.

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STUDY ON ANGULAR RATE INDICATING INSTRUMENT

1. Ion diffusion in various gases to select most promising working fluids.

2. Generation of ions by spark and other methods

3. Methods for pumping working fluid

4. Motion Analysis of A.R.I. and associated equipment

5. Grid geometry and spacing of electrodes

TESTING PROGRAM

1. Gas velocity measurement

2. Gas pumping methods

3. A.R.I. instrument tests

DESIGN OF TEST EQUIPMENT

1. Design of basic rig for gas velocity measurement

2. Design of basic rig for gas pumping tests.

3. Design of basic A.R.I. instrument for test

I.

II.

III.

M A M J J A S O N

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Figure 1.

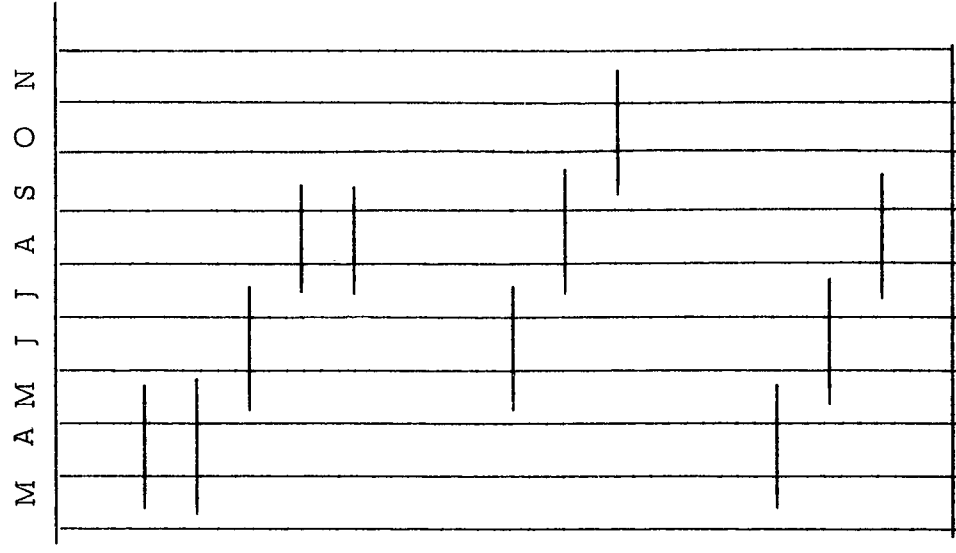|	M	A	M	J	J	A	S	O	N

I. STUDIES ON ANGULAR RATE INDICATING INSTRUMENT

1. Studies Relating to Heat Generation and Measurement

2. Motion Analysis of ARI and Associated Equipment

3. Grid Geometry and Spacing of Electrodes

II. TESTING PROGRAMS

1. Gas Velocity Measurements

2. ARI Instrument Tests

III. DESIGN OF TEST EQUIPMENT

1. Design of Basic ARI Instrument for Test

Figure 2.

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APPENDIX   A

STUDIES PERFORMED ON MIXED ION SYSTEMS

1.1   The Development Laboratories Proposal N-1 suggested that gas

speeds might be accurately measured by injecting or creating low speed

ions into the gas stream and observing the arrival time of these ions at

some down-stream point.   The studies performed to date have led to the

following conclusions:

> 1.   The heavy ion diffusion rates dictate an
> environment near or above normal gas density.   At
> significantly lower density, the diffusion rates
> become large and the pulse or wave shape of the
> group is rapidly distorted.
>
> 2.   If mixed ion systems (heavy positive ions plus
> electrons) are considered, then recombination rates at
> normal gas density are high and it is difficult, maybe
> impossible, to retain a measurable ion group at the
> down-stream observation point.   This observation,
> derived from studies of Xe ions in a gas of normal
> density, is less true of a helium system at 10 atmospheres.
>
> 3.   Detection efficiencies have not been explored
> and raise a major uncertainty in using the above con-
> clusions to make decisions.   Detection systems will be
> somewhat different in mixed ion versus single ion systems.
> Future analytical work should endeavor to establish the
> detection efficiency in single ion systems.
>
> 4.   The concurrent development of a system using
> heated gas pulses detected with a thermocouple is already
> in the experimental stage.   At the present time, this work
> is showing sufficient promise to favor its support instead
> of the originally proposed ion scheme.

1.2   The remainder of Appendix A and also Appendix B describe the theoretical

analysis which have been performed upon ion systems.

MINIMUM MEASURABLE CURRENTS

1.3   The geometry and gas rates are those described in Development
Laboratories proposal N-1.   A gas speed of about 15 cm/second is to be
observed over a transit path of about 0.5 cm by injecting ions and
measuring their speed as they drift with the gas.   The pulses then
arrive at the observation point at 33 millisecond intervals yielding a
fundamental frequency of 30 cps.   If we assume an amplifier input pass
band of 10 to 100 cps and an input resistance of 5 megohms, then the
mean square noise current is:[1]

$$\overline{i}^{2} \quad = \quad \frac{4\,k\,T}{R} \quad . \quad B \qquad\qquad (1)$$

$$B \quad = \quad 90 \quad cps$$

$$R \quad = \quad 5 \times 10^{6} \quad \text{ohms}$$

$$kT \quad = \quad 4.14 \times 10^{-21} \text{ joules at } 300^{\circ} \text{ k}$$

Accordingly, i (rms) = 5.5×10^{-13} amperes.   The corresponding voltage
across the 5 megohm input resistance is 2.75 microvolts.   It may be
noted that although reduction of the input band pass to 0.9 cps will
reduce the noise current ten-fold, that one is thereby inviting electronic
refinements which are not entirely within the spirit of the simple gyro
substitute.

1.4   The ion detector is regarded as a pair of parallel electrodes 1 cm
long of very thin wire and spaced at 0.05 cm apart.   It will be assumed
that the collection aperture is 1 cm by 0.05 cm = 0.05 cm^{2}.   The
detector is instrumented to sense changes in the conductivity of the
gas and it is assumed that all of the positive ions in the aperture are
collected at one electrode and all of the electrons are collected at the
other electrode.   The total current through the electrode systems is then:

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[1]   J.J. Freeman, Principles of Noise, Wiley and Sons, 1958,
        p. 115.

$$i = n_i \, v \, A \, \epsilon \qquad\qquad (2)$$

A = detector area, 0.05 cm^2

v = ion drift speed, 15 cm/sec

ϵ = ion charge, 4.8×10^{-19} coulombs/ion

n_i = ion density (= n_e, the electron density)

1.5   With the additional assumption that the minimum detectable current is of the order of the noise current, the minimum ion-pair density at the detector must be:

$$n_i = \frac{5.5 \times 10^{-13}}{(15)\,(0.05)\,(4.8 \times 10^{-19})} = 1.53 \times 10^6 \text{ ions/cm}^3$$

1.6   In progressing from the point of creation to the detector, ion densities are reduced by diffusion and by recombination.   Whereas diffusional losses are rather easily controlled, recombination rates pose a serious problem to sustaining the minimum detectable current in the above geometry.

ION DIFFUSION (WITHOUT RECOMBINATION)

1.7   It can be shown that the electrons cannot diffuse away from the ions because the electrical potential energies would then greatly exceed the available kinetic thermal energy.   There is some distortion to the diffusion solution in this system, however   we shall first treat the ions as if electrical forces were negligible.   A simple model is conceived in   which ions and electrons are created in equal number in

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a nearly line source geometry which then is dragged into a sheet by the passing gas. This is illustrated in Figure 1 in which the gas velocity is parallel to the time axis, the vertical axis represents the ion density.
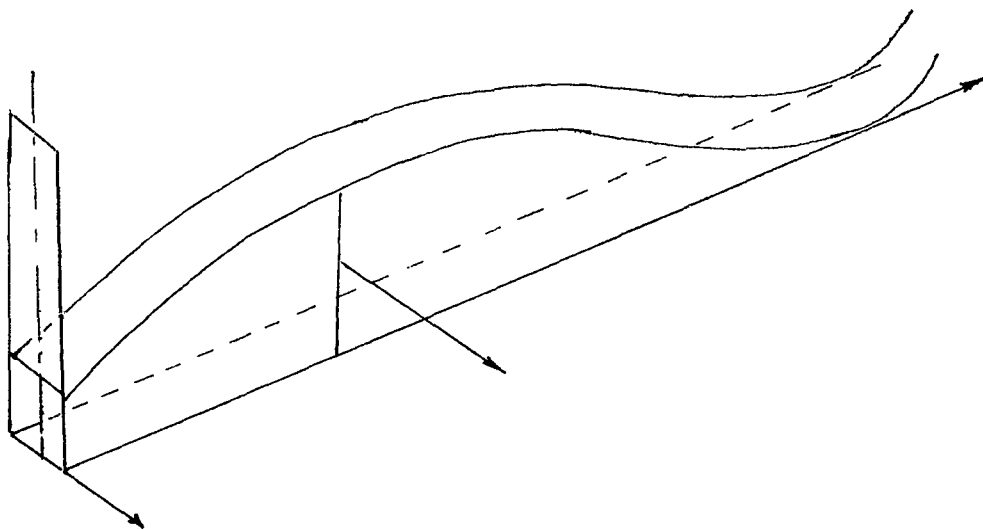
FIGURE I.

The initial width of the sheet is taken as 0.01 cm (2% of the transit path length). The ion density is modulated sinusoidally at about 30 cps. The ion source is conceived as a narrow pencil (0.01 cm diameter) of UV light modulated by a graded disc driven at 1800 rpm.

1.8   As the sheet drifts from its point of creation to the detection point, it spreads laterally by diffusion. The diffusion constant, at normal density (1 atmosphere), for positive xenon ions in xenon is estimated from mobility data to be:[2]

$$D = 1.35 \times 10^{-2} \ cm^{2}/sec$$

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[2]   The mobility data is that of R.N. Varney as reported by S.C. Brown, Basic Data of Plasma Physics, Wiley and Sons, 1959, 0. 72, Figure 3.35. The value of D given above is consistent with $D = \lambda v/3$ and a collision cross-section of $2.1 \times 10^{-14} \ cm^{2}/atom$.

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If one assumes a simple solution of the plane source diffusion equation then the ions are contained between parallel planes at $x = \pm \sqrt{4Dt}$. Assuming one square centimeter of area (normal to the diffusion direction, x), the system volume at time t is:

$$V = 2x = 2\sqrt{4\ Dt} \tag{3}$$

If this volume contains N particles uniformly distributed and if no particles disappear by recombination, the particle density at time t is:

$$n = \frac{N}{V} = \frac{N}{2\sqrt{4\ Dt}} \tag{4}$$

The singularity at $t = 0$ is characteristic of singular source diffusion problems and is removed by computing the time t_o for the plane source to diffuse to the initial finite thickness ($2\ x_o = 0.01$ cm). From (3), this time is found to be 0.463 milliseconds. Accordingly, Equation (4) will be rewritten:

$$n = \frac{N}{2\sqrt{4\ D(t_o+t)}} = n_o \left(\frac{t_o}{t_o + t} \right)^{1/2} \tag{5}$$

where n_o = the initial concentration at creation.

1.9   Table I shows densities 33.3 milliseconds after creation (0.5 cm downstream from the creation point) as a function of initial density and operating pressure.   Operating pressure influences the diffusion constant and hence t_o in accordance with

$$t'_o = \frac{D\ t_o}{D'} = \frac{p't_o}{p} \tag{6}$$

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TABLE I

DENSITY REDUCTION BY DIFFUSION ONLY

| n_o (ions/cm^3) | Ion Pairs/cm^3 after 33.3 milliseconds | | |
	p = 1/2 atm	p = 1 atm	p = 2 atm
5×10^7	3.45×10^5	6.85×10^5	1.35×10^6
5×10^8	3.45×10^6	6.85×10^6	1.35×10^7
5×10^9	3.45×10^7	6.85×10^7	1.35×10^8

It may be noted that (a) the density reduction is strongly influenced and easily controlled by choice of an operating pressure; (b) that final density is directly proportional to initial density; and (c) that the minimum observable density of 1.53×10^6 ions is met with most of the initial densities and pressures of Table 1.

1.10   When recombinations are taken into account, it is found that final density is no longer proportional to initial density and that it becomes more difficult to sustain the minimum observable density.

DIFFUSION WITH RECOMBINATION

1.11   If there are N total particles in volume, V then the density $n = N/V$. The density change with time is given by:

$$\frac{dn}{dt} = - \frac{N}{V^2} \frac{\partial V}{\partial t}\Big|_N + \frac{1}{V} \left(\frac{\partial N}{\partial t}\right)_V \tag{7}$$

The partial derivatives are taken at constant N and constant V respectively. With equation (3) and $n = N/V$, this equation can be written:

$$\frac{dn}{dt} = - \frac{n}{t + t_o} + \frac{1}{V} \left(\frac{\partial N}{\partial t} \right)_V \qquad (8)$$

with $\partial N / \partial t = 0$, this equation has the solution (5). The partial $\partial N / \partial t$ at constant volume is the particle loss due to recombination. It may be expressed by:

$$\frac{\partial N}{\partial t} = V \left(\frac{\partial n}{\partial t} \right)_V = - \alpha V n^2 \qquad (9)$$

The recombination constant, α, is taken as $2 \times 10^{-6} cm^3/sec.$ [3]

Using this equation (9) becomes:

$$\frac{dn}{dt} = - \frac{n}{t + t_o} - \alpha n^2 \qquad (10)$$

As before, the symbol t_o represents the time required for a plane source to diffuse to its initial, finite half thickness, 0.005 cm, and is 0.463 milliseconds for operation at o atmosphere.

1.12   The solution of (4) if n_o is the density at which particles are created is:

$$\frac{1}{n} = \left(1 + \frac{t}{t_o} \right) \left[\frac{1}{n_o} + \alpha t_o \ln \left(1 + \frac{t}{t_o} \right) \right] \qquad (11)$$

The following table shows the particle density at $t = $ 10, 20, and 33.3 milliseconds as a function of n_o when $\alpha = 2 \times 10^{-6}$ and $t_o = 0.463$ milliseconds.

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[3] S.C. Brown, loc. cit, p. 195, Table 8.3. For the source of equation (9)

TABLE II

RESIDUAL ION DENSITIES AT DOWN STREAM OBSERVATION POINTS

n_o	Ion Pairs Per Cm^3		
	$t = 10$ ms	$t = 20$ ms	$t = 33.3$ ms
5×10^7	1.93×10^6	0.98×10^6	0.57×10^6
5×10^8	9.05×10^6	4.10×10^6	2.30×10^6
5×10^9	1.43×10^7	6.24×10^6	3.29×10^6
∞	1.53×10^7	6.45×10^6	3.45×10^6

1.13   It was previously shown that the particle density at the observation point should be 1.53×10^6 ion pairs/cm^3.   The table above shows that although this can be attained, there is not much leeway if times as long as 33.3 milliseconds are involved.   Increasing the pressure appears to be slightly advantageous, this is shown in Table 3 which applies only to observations made at 33.3 milliseconds.   At the higher pressures, recombination is the dominant influence, and at $n_o = \infty$, recombination is the only factor responsible for reducing particle density with time.

TABLE III

EFFECT OF PRESSURE UPON RESIDUAL ION PAIR DENSITY AT 33.3 MILLISECONDS

p (atm)	D (cm^2/sec)	Ion Pairs/cm^3 after 33.3 Milliseconds		
		$n_o = 5 \times 10^7$	$n_o = 5 \times 10^8$	$n_o = 5 \times 10^9$
1/2	2.70×10^{-2}	3.10×10^5	1.60×10^6	2.75×10^6
1	1.35×10^{-2}	5.72×10^5	2.30×10^6	3.29×10^6
2	6.75×10^{-3}	1.01×10^6	3.11×10^6	3.93×10^6
5	2.70×10^{-3}	1.98×10^6	4.44×10^6	5.05×10^6
10	1.35×10^{-3}	3.08×10^6	5.68×10^6	6.19×10^6
50	2.70×10^{-4}	6.91×10^6	9.47×10^6	9.88×10^6
∞	0	1.15×10^7	1.46×10^7	1.50×10^7

1.14   Of particular note is the fact that at initial densities greater than 5×10^8 per cm^3, the residual density at 33.3 milliseconds is very unresponsive to further increases in initial density (compare the last two columns of Table 3). For instance, a computation with $n_o = 5 \times 10^{10}$ yields essentially the same result as that at $n_o = 5 \times 10^9$.

CONCLUSIONS

Because recombinations play a dominant role in limiting the final ion density, the choice of xenon was perhaps unfortunate. Helium with a recombination rate two orders of magnitude lower than xenon, operated at 10 atmospheres to limit diffusion rates, would reduce in density from 5×10^9 to about 9.5×10^7 at 33.3 ms. [4]

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[4] Assuming the diffusion constant is inversely proportional to the square root of the mass and inversely proportional to the operating pressure.

It cannot be concluded from the foregoing that systems with mixed ions cannot be made to work. However, more promising concepts have arisen which are not subject to the recombination limitation. The exploration of these concepts appears to be more fruitful than a continued study of the mixed ion systems.

APPENDIX B

STUDIES PERFORMED ON SINGLE ION SYSTEMS

1.   The studies made of mixed ion systems indicated that ion recombinations would occur rapidly and that the residual ion density at the point of observation might be only marginally observable.   If ions of a single sign are considered (say electrons captured by heavy ions such as bromine) then recombination problems are disposed of.   One implementation of this scheme involves emitting electrons from a thoriated tungsten filament into a gas having a high electron attachment probability.   Although not extensively examined, this method is regarded as a more fruitful line of endeavor than the mixed ion systems.

2.   In the previously considered mixed ion systems, the conventional picture of diffusion was preserved because of the tendency to microscopic electrical neutrality throughout the system.   This is not the case with ions of a single sign and, at least under some conditions, the effect of electrical forces can greatly exceed the diffusion effects.   The analysis of this problem proved to be rather lengthy and is briefly outlined at the end of this Appendix (Note 1).

3.   A system is conceived in which electrons are emitted in a short burst from a heated filament and are almost instantly captured by a gas having high electron affinity.   The negative ions so formed then drift with the main gas stream to a detection grid where their arrival is sensed by the passage of a current in an external circuit.   The minimum detectable current is similar to that computed for the mixed ion system, i.e. 5.5×10^{-13} amperes.   With the electron charge equal to 4.8×10^{-19} coulombs, the

number of ions emitted by the source is:

$$N = \frac{5.5 \times 10^{-13}}{4.8 \times 10^{-19}} \times \frac{1}{30} = 3.82 \times 10^4 \text{ per cycle per cm of filament length.}$$

The ions are formed rapidly and diffuse cylindrically from the region around the filament (See Note 2). The pulse rate is assumed to be 30 cycles per second.

4. It is shown in Note 1 that the quantity $N\epsilon^2/kT$ may be used as a measure of the relative importance of space charge effects and diffusion effects in the expansion of the ion cylinder. At 300° K, $kT = 4.14 \times 10^{-14}$ ergs, $\epsilon^2 = 2.56 \times 10^{-20}$ erg-cm. Hence

$$N\epsilon^2/kT = \frac{(3.82 \times 10^{-4})(2.56 \times 10^{-20})}{4.14 \times 10^{-14}} = 0.0236$$

Using Equation 9 of Note 1, it is apparent that the electrical forces are slight at this initial concentration and that the system can be described by the usual diffusion relations. The initial concentration could be increased 10 fold without seriously distorting the diffusion viewpoint.

5 No data has been found describing the diffusion and/or mobility of halogen ions. Data on 0^- in 0_2 might be expected to be somewhat similar to fluorine and are available. [2] For 0^- in 0_2, the mobility is about 2.5 cm/sec per volt/cm at 1 atmosphere and 273° K. The diffusion constant and mobility are simply related by:

$$\mu = \frac{eD}{kT} \tag{10}$$ [3]

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[2] S.C. Brown, "Basic Data of Plasma Physics," Wiley and Sons, 1959, pp. 68-69

[3] Consistent units for this equation in the cgs system μ, cm^2/sec-statvolt; e, 1.6×10^{-10} esu; k, ergs per degree Kelvin; T in $^\circ$K; and D in cm^2/sec.

This yields $D = 0.176$ cm^2/sec for 0^- in 0_2 and is taken as an estimate of the diffusion constant of F^- in F_2. Fluorine is of interest because of its high election affinity. It is probable that a small amount of a heavier halogen in a heavy inert gas such as xenon would provide a lower, more desirable, diffusion constant. In view of these comments, it is reasonable to believe that a diffusion constant of 0.05 cm^2/sec can be achieved without increasing the system pressure above 1 atmosphere. With such a diffusion constant, the cylinder would expand from a line source to about 0.08/cm radius during its 33 millisecond transit time to the detector and its arrival at the detection grid would occur over a time interval of about 10 milliseconds. The detection grid should be on the order of 0.16 cm wide in order to capture a substantial portion of the negative ions transmitted from the source.

6.   Because of the finite distance the electrons travel prior to attachment, the initial configuration of diffusing, negative ions, is finite. However, the rough calculation of Note 2 indicates that a few per cent of halogen in xenon will attach 90% of the initial electrons in a radius which is smaller than the diffusion distance (0.08 cm) estimated above.

7.   The foregoing scheme using negative ions appears to be flexible, composition and pressure can be varied to control diffusion, and the scheme does not have the inherent limitation imposed by recombination in the mixed ion case.

NOTE 1:   The Effect of Space Charge Upon Diffusion Rate

A general equation for diffusion with forces was derived by M.H. Johnson[1]. With the aid of hydrodynamic continuity, assuming isothermal ($p = nkT$) behavior and irrotational flow, and neglecting the streaming kinetic energy and $\partial^2 n / \partial t^2$, it is possible to reduce this equation in the case of cylindrical symmetry to

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial n}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{r n F}{kT} \right) + \frac{1}{D} \frac{\partial n}{\partial t} \tag{1}$$

Here n is the particle density at radius r and time t, F is the force acting on each particle, D is the diffusion constant, T the temperature and k is Boltzmann's constant.

The force acting on a particle at r is due to the charge interior to r and may be written:

$$F = \frac{4 \pi \epsilon^2}{r} \int_o^r n(r') \, r' \, dr' \tag{2}$$

where ϵ is the electronic charge (it is assumed that each charged particle is only singly charged).

Inserting Equation (2) in (1) the equation for diffusion with space charge is:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial n}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left[\frac{4 \pi \epsilon^2}{kT} n \int_o^r n(r') \, r' \, dr' \right] + \frac{1}{D} \frac{\partial n}{\partial t} \tag{3}$$

---

[1] See the summary in Chapman and Cowling, Mathematical Theory of Non-Uniform Gases, Second Edition, Cambridge University Press, p. 415, Equation 1.

It will be seen that if the particle density is low enough (or if the electronic charge ϵ is zero) then the equation reduces to the usual simple diffusion equation.

A solution of Equation 3 was desired for an instantaneous line source in an infinite medium at time $t = 0$. It can be shown that for r very small a solution of (3) is:

$$n = \frac{N\, e^{-r^2/4\,\alpha\, t}}{4\,\pi\,\alpha\, t} \tag{4}$$

where N = the number of ions/cm introduced by the line source and:

$$\alpha = D\left(1 + \frac{N\,\epsilon^2}{kT}\right) \tag{5}$$

This solution approaches the usual line source diffusion solution if $N\,\epsilon^2/kT$ approaches zero. The magnitude of $N\,\epsilon^2/kT$ is a measure of the effect of electrical forces in magnifying the diffusion constant D.

An approximate solution, applicable to all values of r and t is:

$$n = \frac{N}{m+1} \cdot \frac{1}{4\,\pi\, D\, t}\left(1 + x + \frac{x^2}{2} \cdots \frac{x^m}{m!}\right)\, e^{-x} \tag{6}$$

$$x = \frac{m}{N\epsilon^2/kT} \cdot \frac{r^2}{4\,Dt}$$

The differential equation does not impose any restriction upon the choice of m, however if m is chosen as

$$m = N\,\epsilon^2/kT \tag{7}$$

then the solution at $r = 0$ is the same as that given by (4) and (5). Further, if $N \longrightarrow 0$, the usual diffusion solution is obtained. In general,

$N \epsilon^2 / kT$ will not be an integer and the last term of the polynomial should be chosen with m equal to that integer closest to $N\epsilon^2 / kT$.

The solution expressed by (6) is very flat out to $x \simeq m$. In the interior region, the space charge effect has produced a nearly uniform particle density. The point at which the diffusion front is the steepest is:

$$r^2 = 2 \left(1 + \frac{2 N \epsilon^2}{kT} \right) Dt \tag{8}$$

Although this is one way of specifying the extent of the diffusion, an alternative way is by computing the mean square diffusion distance:

$$\overline{r^2} = 2 \left(2 + \frac{N \epsilon^2}{kT} \right) Dt \tag{9}$$

Equations (8) and (9) are to be regarded as useful approximations. They yield the same result only if $N \epsilon^2 / kT = 1$. In the event that $N \epsilon^2 / kT < 1$, the spreading is controlled primarily by diffusion and (9) is a natural extension of the usual diffusion distance, $r^2 = 4Dt$. If $N \epsilon^2 / kT > 1$, then the diffusion-like solution becomes subordinate to the space charge effect and (8) gives a better physical representation of the radius which contains a substantial fraction of the particles. In this case the distribution is fairly uniform over most of the confining radius.

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NOTE 2:   Mean Distance for Electron Attachment to Halogens.

The data of R.H. Healey [4] indicates an attachment cross-section of about 10^{-18} cm^2 for 1 volt electrons in Cl_2 and Br_2 and perhaps 3 times this value for the attachment of low energy electrons (say 0.5 ev or less) to I_2. The data does not clearly indicate what cross-sections might obtain for electron energies at lower energies. It will be assumed that the attachment cross-section remains at 10^{-18} cm^2 for electrons down to zero energy, that the electrons which leave the filament are rapidly scattered by an inert gas to thermal temperature $(300^O K)$ where the mean electron speed is 1.08×10^7 cm/second.

The flux of electrons is $\phi = nv$ and ϕ satisfies the diffusion equation. [5]

$$\frac{\lambda_s}{3} \nabla^2 \phi - \Sigma_a \phi + S = \frac{\partial n}{\partial t} = \frac{1}{v} \frac{\partial \phi}{\partial t}$$

Here λ_s is the scattering mean free path, Σ_a is the absorption (attachment) microscopic cross-section, S is the source strength and v is the electron speed. Assuming that the gas is largely xenon, then the electron scattering cross-section in xenon at 1 atmosphere is not less than 3800 cm^2/cm^3. [6] Hence, the scattering mean free path is not greater than $1/3800 = 2.63 \times 10^{-4}$ cm. If the total gas is made up of xenon containing a small fraction, k, of halogen, then the absorption cross-section is:

$$\Sigma_a = (2.44 \times 10^{19}) \ (10^{-16}) \ k$$

$$= 24.4 \ k \quad cm^2/cm^3.$$

$\underline{(2.44 \times 10^{19}} = $ gas particle density at 1 atmosphere and $300^O K).$

[4] As reported by S.C. Brown, loc. cit, pg. 178

[5] Glasstone and Edlund "Elements of Nuclear Reactor Theory", Van Nostrand, 1952, p. 101, Equation 5.35.1.

[6] See S.C. Brown, loc. cit, Figure 1.10 and the remarks following Equation 1.4.

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We are primarily concerned with the mean distance from the origin at which electrons become attached and so consider the steady state situation described by:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} - \frac{3 \Sigma_a}{\lambda_s} \phi = 0$$

This equation has the solution:

$$\phi = a \, Kh_o \, (\beta r)$$

where Kh_o is the Bessel function $i \, H_o^{(1)} \, (i \, x)$ tabulated by Janke and Emde, Fourth Edition, page 236. The function is positive everywhere, infinite at the origin, and decreases in exponential-like fashion with increasing radius. The factor $\beta = \sqrt{3 \Sigma_a / \lambda_s} = 528 \sqrt{k}$ with the previously given values of Σ_a and λ_s. The solution is normalized by requiring that the integral of $\Sigma_a \phi$ over all space (i.e. the total rate of electron attachment), is equal to the rate at which electrons are admitted at the source.

$$N = \int \Sigma_a \phi \, dv = 2\pi a \Sigma_a \int_o^\infty Kh_o \, (\beta r) \, r \, dr$$

Integrating as indicated,

$$a = \beta^2 \, N / 4 \Sigma_a = (3/4) \, N / \lambda_s$$

The normalized solution is then:

$$= \frac{3}{4} \, \frac{N}{\lambda_s} \, Kh_o \, (\beta r)$$

Although some electrons continue as r becomes large, the decay of electron flux is very rapid. The number of electrons absorbed interior to r is:

$$n = 2\pi \Sigma_a \int_o^r \phi \, r \, dr$$

$$= \frac{\pi N}{2} \left[\frac{2}{\pi} - \beta r \, Kh_1 (\beta r) \right.$$

Therefore: $\quad \dfrac{n}{N} = 1 - \dfrac{\pi}{2} \beta r \, Kh_1 (\beta r)$

For $n/N = 0.9$ (i.e., 90% of the electrons have been absorbed), βr must satisfy

$$\beta r \, Kh_1 (\beta r) = (0.1)(2/\pi) = 0.0636$$

$$\text{or} \quad \beta r = 3.22$$

with $\beta = 528 \sqrt{k}$, the radius r within which 90% of the electrons become attached is given by $r = 3.22/528 \sqrt{k}$. This radius is shown as a function of k in the following table:

k	r (cm)
0.01	0.0610
0.02	0.0431
0.04	0.0305
0.10	0.0194

STUDIES PERFORMED ON THE THERMAL SYSTEM

1.   The thermal system consists of a pulsed heater and a down-
stream detector which senses the arrival of pulses of heated gas. The
detector may be a thermocouple or a resistance thermometer, the heater
is a simple wire. The heater and sensors must be small enough to have
little thermal inertia at the operating frequency (30 cps). The transit
time of the gas pulse must be sufficiently short that the gas pulse does
not seriously decay by thermal conduction between creation and detection.

2.   The calculations cf this appendix show how the important dimensions
(transit time and wire size) are established.

3.   The problem of the behavior of the heat pulse as it transits from
the heater to the detector is equivalent to the problem of a heat source
moving through a fixed fluid. This problem is treated in the literature [1]
and the equation for the temperature of the fluid may be written:

$$\frac{\partial \Theta}{\partial t} = \alpha \nabla^2 Q + \frac{Q(t)}{\rho c} \cdot \delta(x - vt) \cdot \delta(y) \tag{1}$$

Specifically, this relates to a line source of power Q (t) per unit length
which moves at speed v through a fixed medium. The temperature at
x, y, and t is Θ, α is the thermal diffusivity, ρ c is the volumetric
heat capacity and the delta functions indicate that the source is always

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[1] See Sneddon "Fourier Transforms" McGraw-Hill, 1951, p. 201

located at $y = 0$, $x = vt$. If the heating wire is driven by a sinusoidal voltage, then the source term can be written:

$$Q(t) = Q^1 \sin^2 \omega t = Q^1 (1 - \cos 2\omega t)$$

where $Q^1 = E^2/R$ and E is the peak driving voltage. The solution of Equation 1 is:

$$\Theta(x,y,t) = \frac{Q^1}{8\pi\alpha\rho c} \int_0^t (1 - \cos 2\omega\tau) \exp$$

$$\left[-\frac{(x - v\tau)^2 + y^2}{4\alpha(t-\tau)} \right] \frac{d\tau}{t-\tau} \tag{2}$$

In the present problem, we are more concerned with the solution as one follows the source at a fixed distance $\xi = vt - x$. The The integral is approximated by the method of stationary phase, so an approximate, algebraic solution for the case with $y = 0$ is:

$$\Theta(\xi) = \frac{Q}{8\rho c \sqrt{\pi\alpha\xi v}} \cdot \left[1 - \cos 2\omega\left(\frac{vt - \xi}{v}\right) \right] \tag{3}$$

and, if $y \neq 0$, then

$$\frac{\Theta(\xi, y)}{\Theta(\xi, 0)} = \left[1 - \left(\frac{y}{\xi}\right)^2 \right] e^{-vy^2/4\alpha\xi} \tag{4}$$

The nature of the approximation is such that the solution is only valid in the region behind the source and only if the driving frequency, $f = \omega/2\pi$ satisfies:

$$f < f_c = \frac{v}{8}\sqrt{\frac{v}{\alpha\xi}} \tag{5}$$

In the present case, the value of the right-hand side of (5) is about 38 cps, whereas the contemplated operating frequency is 20 to 30 cps.

4. The solution (3) shows that at distance ξ from the source, the temperature oscillates between zero and peak values given by:

$$\Theta_{max} (\xi) = \frac{Q^1}{4 \rho c \sqrt{\pi \propto \xi}}$$ (6)

The thermal diffusivity, $\propto$, is taken as 0.093 cm^2/sec; the volumetric heat capacity, ρc, is 1.33×10^{-4} cal/cm^3 - $^\circ$K, and the speed v is 16.35 cm/sec. In order to obtain a peak temperature rise of 1°C at ξ 0.5 cm from the source, the peak source strength (from Equation 6 must be 8.2×10^{-4} cal/sec. or 3.42×10^{-3} watts per cm of heater length. With this power input, the peak temperatures observed at other points ξ, y are shown in the table below:

	$\Theta (\xi, y)$	$^\circ$C	
y	$\xi = 0.32$	0.40	0.50
0	1.25	1.12	1.00
0.02	1.18	1.07	0.97
0.04	0.99	0.93	0.86
0.06	0.75	0.74	0.72

(The dimensions ξ and y are centimeters).

5. The frequency restriction implied by (5) is somewhat arbitrary since it does not quantitatively specify the errors arising when the equality obtains. For this reason, numerical integrations were performed using an operating frequency of 25.5 cps. The temperature fluctuation (peak-to-trough) at $\xi = 0.32$ and y = 0 was found to be 1.19 degrees instead of the 1.25 degrees of the preceding table. This indicates that the approximate solution is quite good in the region of interest

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(20 to 30 cps). It should be noted that if the heater driving voltage is alternating at 25 cps, then the actual heat input is varying at a frequency of 50 cps. For this reason, the criterion (5) may be interpreted as indicating that serious attenuation of signal frequencies should not be expected below 38 cps. The numerical integration indicates that signal frequencies up to 51 cps are propagated without serious attenuation for the contemplated system parameters.

Although the foregoing establishes the fact that low pressure gases can be satisfactory from the standpoint of thermal conductivity, it has been found that they are unsatisfactory from a heat transfer view point. Specifically, they do not remove heat rapidly enough from the heater win and so lead to an undesirably long thermal pulse (< 50 milliseconds). Calculations indicate that liquids and/or high density gases should be satisfactory in both respects and current activities are devoted to investigations of liquid systems.